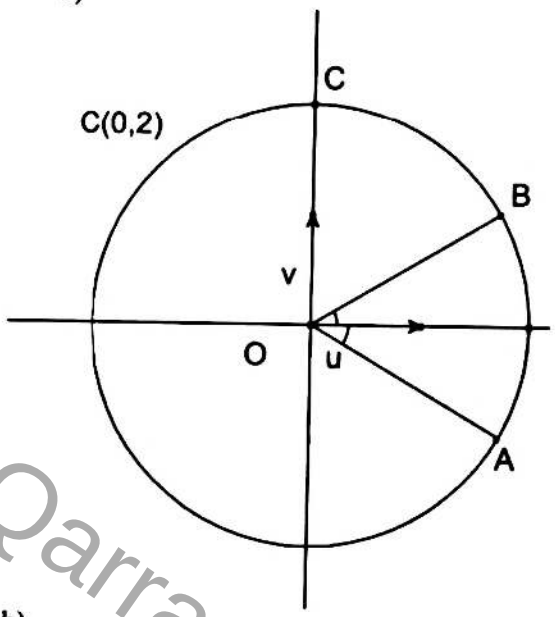
4) $z_A = \sqrt{3} - i$ $z_B = \sqrt{3} + i$ $z_C = 2i$

a)



b)

* $aff\left(\overrightarrow{OC}\right) = z_C = 2i$ et $aff\left(\overrightarrow{AB}\right) = z_B - z_C = 2i$

D'où $\overrightarrow{OC} = \overrightarrow{AB}$ $\Rightarrow$ OABC est un parallélogramme (1)

* $OA = |z_A| = 2$ $OC = |z_C| = 2$ $\Rightarrow \boxed{OA = OC}$ (2)

(1)+(2) $\Rightarrow$ OABC est un losange.

EX 37 :

I) $\alpha \in \left[0, \frac{\pi}{2}\right]$

1) $\left(e^{2i\alpha} + 2e^{i\alpha}\right)^2 - \left(e^{2i\alpha} - 2e^{i\alpha}\right)^2 = 8e^{3i\alpha}$

2) $z^2 - \left(e^{2i\alpha} + 2e^{i\alpha}\right)z + 2e^{3i\alpha} = 0$

$\Delta = \left(e^{2i\alpha} + 2e^{i\alpha}\right)^2 - 8e^{3i\alpha} = \left(e^{2i\alpha} - 2e^{i\alpha}\right)^2 \rightarrow \delta = e^{2i\alpha} - 2e^{i\alpha}$

Les solutions sont :

$z' = \frac{\left(e^{2i\alpha} + 2e^{i\alpha}\right) - \left(e^{2i\alpha} - 2e^{i\alpha}\right)}{2} = 2e^{i\alpha}$ et $z'' = \frac{\left(e^{2i\alpha} + 2e^{i\alpha}\right) + \left(e^{2i\alpha} - 2e^{i\alpha}\right)}{2} = e^{2i\alpha}$

II) $z_A = e^{2i\alpha}$; $z_B = 2e^{i\alpha}$; $z_{A'} = ie^{2i\alpha}$; $z_{B'} = -2ie^{i\alpha}$

1)

a) $z_{A'} = e^{i\frac{\pi}{2}} e^{2i\alpha} = e^{i(\frac{\pi}{2} + 2\alpha)}$ et $z_{B'} = 2e^{-i\frac{\pi}{2}} e^{i\alpha} = 2e^{i(\alpha - \frac{\pi}{2})}$

Qarray.tn - Aqra Blech - Qarray.tn